PUNJAB PUBLIC SERVICE COMMISSION

WRITTEN TEST FOR RECRUITMENT TO THE POST OF
LECTURER MATHEMATICS (BS-17)
(Male/ Female)
IN THE PUNJAB HIGHER EDUCATION DEPARTMENT-2

Paper Code

ROLL NO

Q.1. $\int_{-4}^{0} \frac{t\,dt}{\sqrt{16-t^2}} =$ _____.
(A) 0 **(B)** Divergent **(C)** -4 **(D)** 4

Q.2. The period of the function $A\cos\omega t + B\sin\omega t$ is _____.
(A) $\frac{\omega}{2\pi}$ **(B)** $2\pi\omega$ **(C)** $\frac{\omega}{2\pi}$ **(D)** $\frac{2\pi}{\omega}$

Q.3. $A = (-4x-3y+az)\hat{\imath} + (bx+3y+5z)\hat{\jmath} + (4x+cy+3z)\hat{k}$ is irrotational when a, b, c are _____.
(A) 4, -3, 5 **(B)** 4, 5, -3 **(C)** -3, 4, 5 **(D)** 2, 3, 5

Q.4. $V = (-4x-6y+3z)\hat{\imath} + (-2x+y-5z)\hat{\jmath} + (5x+6y+az)\hat{k}$ is solenoidal for $a =$ _____.
(A) 1 **(B)** 2 **(C)** 3 **(D)** 4

Q.5. $\int_{(0,0)}^{(2,1)} (10x^4 - 2xy^3)\,dx - 3x^2y^2\,dy$ along the path $x^4 - 6xy^3 = 4y^2$ is _____.
(A) 56 **(B)** 60 **(C)** 62 **(D)** 64

Q.6. If S is the closed surface and v is the volume enclosed by S then $\iint_S \vec{r}\cdot\vec{n}\,ds =$ _____.
(A) v **(B)** 2v **(C)** 3v **(D)** 4v

Q.7. Centrifugal acceleration is _____.
(A) $-\omega\times(\omega\times r)$ **(B)** $\omega\times(\omega\times r)$ **(C)** $\omega\cdot(\omega\times r)$ **(D)** $r\times(\omega\times r)$

Q.8. Number of degrees of freedom of two particles connected by a rigid rod moving freely in a plane is _____.
(A) 2 **(B)** 3 **(C)** 4 **(D)** 5

Q.9. The centroid of a uniform semicircular wire of radius a is _____.
(A) $2a/\pi$ **(B)** $4a/\pi$ **(C)** a/π **(D)** $a/2\pi$

Q.10. Moment of inertia of a rectangular plate with sides a, b about an axis $\perp$ to plate and passing through vertex is _____.
(A) $\frac{1}{3}Ma^2$ **(B)** $\frac{1}{3}Mb^2$ **(C)** $\frac{1}{3}M(a^2-b^2)$ **(D)** $\frac{1}{3}M(a^2+b^2)$

Q.11. Every bounded infinite set has at least one limit point, is the statement of _____.
(A) Hein-Borel Theorem **(B)** Weierstrass-Bolzano Theorem **(C)** Cantor's Intersection Theorem **(D)** None of these

Q.12. $\lim_{z\to 0}\frac{\bar{z}}{z} =$
(A) $\frac{1+i}{1-i}$ **(B)** 1 **(C)** Does not exist **(D)** -1

Q.13. Cauchy-Riemann equations in polar form are _____.
(A) $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial\theta}, \frac{\partial v}{\partial r} = \frac{-1}{r}\frac{\partial u}{\partial\theta}$ **(B)** $\frac{\partial u}{\partial r} = \frac{-1}{r}\frac{\partial v}{\partial\theta}, \frac{\partial v}{\partial r} = \frac{1}{r}\frac{\partial u}{\partial\theta}$ **(C)** $\frac{\partial u}{\partial r} = \frac{-1}{r}\frac{\partial v}{\partial\theta}, \frac{\partial v}{\partial r} = \frac{-1}{r}\frac{\partial u}{\partial\theta}$ **(D)** $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial\theta}, \frac{\partial v}{\partial r} = \frac{1}{r}\frac{\partial u}{\partial\theta}$

Q.14. Evaluate $\int_C \frac{z^2-z+1}{z-1}\,dz$, where C is the circle $|z| = \frac{1}{2}$:
(A) 1 **(B)** 2 **(C)** $\frac{1}{2}$ **(D)** 0

Q.15. The principal value of $(-i)^i$ is:
(A) $e^{-\pi/2}$ **(B)** 1 **(C)** $e^{\pi/2}$ **(D)** e^{π}

Q.16. The Residue of $f(z) = \frac{z^2-2z}{(z+1)^2(z^2+4)}$ at $z = 2i$ is _____.
(A) $\frac{14}{25}$ **(B)** $\frac{7+i}{25}$ **(C)** $\frac{7-i}{25}$ **(D)** $\frac{-7-i}{25}$

Q.17. Radius of convergence of $\sum(3+4i)^n z^n$ is _____.
(A) $\frac{1}{5}$ **(B)** 5 **(C)** 7 **(D)** ∞

Q.18. $\lim_{n\to\infty}\left(1+\frac{z}{n}\right)^n$ is _____.
(A) 1 **(B)** 0 **(C)** e^z **(D)** e^n

Q.19. $U(x,y) = e^x\cos y$ is _____.
(A) Harmonic **(B)** Analytic **(C)** Not Harmonic **(D)** None of these

Q.20. $\int_0^{\infty}\frac{\sin x}{x}\,dx =$ _____.
(A) 0 **(B)** $-\frac{\pi}{2}$ **(C)** $\frac{\pi}{2}$ **(D)** π

Q.21. $\mathrm{Log}(1+i) =$ _____.
(A) $\frac{1}{2}\ln 2 + \frac{\pi i}{4}$ **(B)** $\frac{1}{2}\ln 2 - \frac{\pi i}{4}$ **(C)** $\frac{1}{2}\ln 2 - \frac{3\pi i}{4}$ **(D)** $\frac{1}{2}\ln 2 + \frac{3\pi i}{4}$

Q.22. Which of the following space is complete.
(A) Q **(B)** $]0,1]$ **(C)** Z **(D)** R

Q.23. Least upper bound of $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots\right\}$ is:
(A) 1 **(B)** 0 **(C)** ∞ **(D)** $\frac{n}{n+1}$

Q.24. $\lim_{x\to 1}\frac{x^x - x}{1-x+\ln x}$ is:
(A) 2 **(B)** -2 **(C)** 1 **(D)** -1

Q.25. $\lim_{x\to 0} x^{\sin x}$ is _____.

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(A) 0 **(B)** $\frac{1}{2}$ **(C)** e **(D)** ∞

Q.26. Minimum and maximum values of $f(x) = x^{2/3}(x^2 - 8)$ in interval $[-1, \frac{1}{2}]$ are
(A) -7, 0 **(B)** 0, 6 **(C)** 1, 2 **(D)** -2, 3

Q.27. $\int_0^1 \frac{4}{1+x^2}\, dx =$ _____.
(A) 0 **(B)** π **(C)** $\frac{4\pi}{3}$ **(D)** $-\pi$

Q.28. $\int_0^{\pi} \csc^2 x\, dx =$ _____.
(A) 0 **(B)** 1 **(C)** -1 **(D)** ∞

Q.29. $\lim_{x\to 0} \sin\frac{1}{x} =$ _____.
(A) does not exist **(B)** 1 **(C)** 0 **(D)** -1

Q.30. $\int_0^{3\pi/4} |\cos x|\, dx =$ _____.
(A) $\frac{1}{\sqrt{2}}$ **(B)** $\frac{-1}{\sqrt{2}}$ **(C)** ∞ **(D)** $2 - \frac{1}{\sqrt{2}}$

Q.31. $\sec\left(\tan^{-1}\frac{2}{3}\right) =$ _____.
(A) $\frac{2}{\sqrt{13}}$ **(B)** $\frac{3}{\sqrt{13}}$ **(C)** $\frac{\sqrt{13}}{3}$ **(D)** $\frac{\sqrt{13}}{2}$

Q.32. Which of the following is convergent series?
(A) $\sum \frac{1}{n^2}$ **(B)** $\sum \frac{1}{\sqrt{n}}$ **(C)** $\sum \frac{1}{n}$ **(D)** $\sum \frac{1}{n^{1/5}}$

Q.33. $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$ is the Maclaurin's series of _____.
(A) $\cos x$ **(B)** $\sin x$ **(C)** $\sinh x$ **(D)** $\cosh x$

Q.34. $\int_1^2 \int_0^{y^{3/2}} \frac{x}{y^2}\, dx\, dy =$ _____.
(A) $\frac{3}{4}$ **(B)** $\frac{7}{8}$ **(C)** $\frac{3}{2}$ **(D)** $\frac{1}{2}$

Q.35. Domain of $f(x) = \sqrt{1-x^2}$ is _____.
(A) $x < 1$ **(B)** $x > 1$ **(C)** $|x| \le 1$ **(D)** $|x| \ge 1$

Q.36. Domain of $f(x) = \frac{1}{\sqrt{(1-x)(2-x)}}$ is _____.
(A) $R \setminus \{1, 2\}$ **(B)** $R \setminus \{1, 2\}$ **(C)** $[1, 2]$ **(D)** $]1, 2[$

Q.37. $f : R \to (-1, 1)$ defined by $f(x) =$ _____ is bijective.
(A) $\frac{x}{1-|x|}$ **(B)** $\frac{x}{1+|x|}$ **(C)** $\frac{1}{1+|x|}$ **(D)** $\frac{x}{-1+|x|}$

Q.38. Interval of convergence of $\sum_{k=1}^{\infty} x^k$ is _____.
(A) $]-1, 1[$ **(B)** $[-1, 1]$ **(C)** $(-\infty, +\infty)$ **(D)** $x = 0$

Q.39. Which of the following are open in the usual metric space (R, d)?
(A) Subsets of R **(B)** Union of open intervals **(C)** Intervals **(D)** Singleton subsets

Q.40. Let $A = (0, 1] \cup (1, 3)$ and R with usual metric space. Then $A^0 =$ _____.
(A) $A \setminus \{0\}$ **(B)** $A \setminus \{1\}$ **(C)** $A \setminus \{3\}$ **(D)** $(0, 1) \cup (1, 3)$

Q.41. Let A be a finite subset of a metric space X. Then $A^d =$ _____.
(A) singleton set **(B)** $\varnothing$ **(C)** A **(D)** $X \setminus A$

Q.42. Let A be a finite subset of (X, d). Then A is _____.
(A) Open set **(B)** Open as well as closed **(C)** Closed set **(D)** neither open nor closed

Q.43. If Y is a subset of (X, d) then _____.
(A) Every open set in Y is open in X. **(B)** Every open set in X is open in Y. **(C)** O is open in $Y \Leftrightarrow O$ is open in X **(D)** O is open $\Leftrightarrow O = Y \cap G$ where G is open in X.

Q.44. Let $f(x) = 1 + x^3$. Then $(0, 0)$ is the point of _____.
(A) maximum value **(B)** minimum value **(C)** point of inflection **(D)** none of these

Q.45. Number of elements in a co-finite topological space (X, τ) where $X = \{s, t, u\}$ is _____.
(A) 2 **(B)** 3 **(C)** 4 **(D)** 8

Q.46. The boundary of a subset $B = \left\{\frac{1}{n} : n \in N\right\}$ of (R, d) is _____.
(A) B **(B)** $\{0\}$ **(C)** $B \cup \{0\}$ **(D)** $\varnothing$

Q.47. The real line R is homeomorphic to _____.
(A) $(0, 4)$ **(B)** $[-1, 1]$ **(C)** Q **(D)** Z

Q.48. R with co-finite topology is _____.
(A) T_0-space **(B)** T_1-space **(C)** T_1-space but not T_2-space **(D)** T_2-space

Q.49. Let $X = \{a, b, c\}$, $\tau = \{\varnothing, \{a\}, \{b\}, \{a, b\}, X\}$. Then X is _____.
(A) T_1-space **(B)** Regular space **(C)** T_2-space **(D)** Normal space

Q.50. Which of the following is connected in R with usual topology?
(A) N **(B)** Q **(C)** $(0, 1]$ **(D)** Z

Q.51. Which of the following topology is not totally disconnected?
(A) $\{1\}$ **(B)** Discrete space **(C)** R with usual topology **(D)** Q

Q.52. Which of the following is nowhere dense in R _____.
(A) $R \setminus Z$ **(B)** Z **(C)** $\bigcup_{n\in Z}(n, n+1)$ **(D)** Q

Q.53. Which of the following is dense in R _____.
(A) N **(B)** Z **(C)** $R \setminus Z$ **(D)** Q

Q.54. $xy'' + y' = 0$ has a solution $y = \ln x$ on interval _____.
(A) $(0, \infty)$ **(B)** $(-\infty, 0)$ **(C)** $(-\infty, \infty)$ **(D)** $[0, \infty[$

Q.55. Which of the following is not linear?
(A) $y' = (\sin x)y$ **(B)** $y' = (\sin y)x + e^x$ **(C)** $y' + xy = e^x y$ **(D)** $y' = 5$

Q.56. Solution of $y' = \dfrac{x+y}{x}$ is _____.
(A) $y = \ln|kx|$ **(B)** $y = \ln|x|$ **(C)** $y = x\ln|kx|$ **(D)** $y = \ln|x| + k$

Q.57. Which of the following differential equation is not exact?
(A) $2xy\,dx + (1+x^2)\,dy = 0$ **(B)** $y\,dx - x\,dy = 0$ **(C)** $y' = \dfrac{2 + ye^{xy}}{2y - xe^{xy}}$ **(D)** $(x + \sin y)\,dx + (x\cos y - 2y)\,dy$

Q.58. Integrating factor for $y' + \left(\dfrac{4}{x}\right)y = x^4$ is _____.
(A) x^4 **(B)** $\ln x^4$ **(C)** $4\ln|x|$ **(D)** $\ln|x|$

Q.59. The area bounded by $y = 4 - x^2$ and X-axis is _____.
(A) $\frac{4}{3}$ **(B)** $\frac{8}{3}$ **(C)** $\frac{16}{3}$ **(D)** $\frac{32}{3}$

Q.60. Which of the following is scalar?
(A) $(\vec{a}\cdot\vec{b})\vec{c}$ **(B)** $\vec{a}\cdot(\vec{b}\times\vec{c})$ **(C)** $\vec{a}\times(\vec{b}\times\vec{c})$ **(D)** $(\vec{a}\cdot\vec{b})(\vec{a}-\vec{a})$

Q.61. Projection of $\vec{a}$ on $\vec{b}$ is _____.
(A) $\vec{a}\cdot\vec{b}$ **(B)** $\dfrac{\vec{a}}{|\vec{a}|}\cdot\vec{b}$ **(C)** $\vec{a}\cdot\dfrac{\vec{b}}{|\vec{b}|}$ **(D)** $\vec{a}\times\vec{b}$

Q.62. Which of the following is scalar quantity?
(A) Momentum **(B)** Magnetic field intensity **(C)** Specific heat **(D)** Moment of force

Q.63. A vector lying in the plane of $\vec{a}$ and $\vec{b}$ is _____.
(A) $(\vec{a}\times\vec{b})\times\vec{c}$ **(B)** $\vec{a}\times(\vec{b}\times\vec{c})$ **(C)** $(\vec{c}\times\vec{a})\times\vec{b}$ **(D)** $(\vec{c}\times\vec{b})\times\vec{a}$

Q.64. Let $\vec{t}, \vec{n}$ and $\vec{b}$ denote respectively the tangent, principal normal and binormal vectors to the curve. The osculating plane to the curve at P contains _____.
(A) $\vec{t}, \vec{b}$ **(B)** $\vec{n}, \vec{b}$ **(C)** $\vec{t}, \vec{n}$ **(D)** $\vec{t}, \vec{n}, \vec{b}$

Q.65. Let $\vec{t}, \vec{n}, \vec{b}$ be as in the above question. Then $\tau\vec{b} - k\vec{t} =$ _____.
(A) $\dfrac{d\vec{t}}{ds}$ **(B)** $\dfrac{d\vec{n}}{ds}$ **(C)** $\dfrac{d\vec{b}}{ds}$ **(D)** $\dfrac{d}{ds}\left(\dfrac{\vec{t}\times\vec{n}}{ds}\right)$

Q.66. Normal plane is perpendicular to _____.
(A) $\vec{t}$ **(B)** $\vec{n}$ **(C)** $\vec{b}$ **(D)** $\vec{t}\times\vec{n}$

Q.67. $\vec{t}\times\vec{b} =$ _____.
(A) $\vec{n}$ **(B)** $-\vec{n}$ **(C)** $\vec{n}\times\vec{b}$ **(D)** none of these

Q.68. $\{x \mid x \in C : x^4 = 1\}$ is a _____.
(A) Subgroup of $(C \setminus \{0\}, \cdot)$ **(B)** Subgroup of $(C, +)$ **(C)** Non cyclic group **(D)** Subgroup of $(Q \setminus \{0\}, \cdot)$

Q.69. R^3 under vector product forms a _____.
(A) group **(B)** monoid **(C)** semi-group **(D)** groupoid

Q.70. An element x of group G satisfying $x^2 = x$ is called _____.
(A) Involution **(B)** Idempotent **(C)** Transposition **(D)** Cycle

Q.71. $\dfrac{Z}{\langle n\rangle}$ is isomorphic to _____.
(A) nZ **(B)** $\langle n\rangle$ **(C)** Z_n **(D)** $\{0, \pm 2n, \pm 4n, \ldots\}$

Q.72. Let $G = \langle a : a^{12} = e\rangle$. Then $G =$ _____.
(A) $\langle a^5\rangle$ **(B)** $\langle a^6\rangle$ **(C)** $\langle a^2\rangle$ **(D)** $\langle a^8\rangle$

Q.73. Let $G = \langle b : b^{17} = e\rangle$. Then G can be generated by _____.
(A) Any element of G **(B)** Any non identity element of G **(C)** b, b^{-1} are the only generators of G **(D)** Identity

Q.74. If $G = \langle \alpha, \beta : \alpha^3 = \beta^2 = (\alpha\beta)^2 = e\rangle$ then $N_G(\{e, \beta\}) =$ _____.
(A) $\{e\}$ **(B)** $\{e, \beta, \alpha\beta\}$ **(C)** G **(D)** $\{e, \beta\}$

Q.75. Let $G = \langle \alpha, \beta : \alpha^4 = \beta^2 = (\alpha\beta)^2 = e\rangle$. Then $Z(G) =$ _____.
(A) $\{e\}$ **(B)** $\{e, \alpha^2\}$ **(C)** $\{e, \alpha, \alpha^2, \alpha^3\}$ **(D)** G

Q.76. Which of the following is not true for an Abelian group G.?

(A) $[a,b]=e\ \forall a,b\in G$ **(B)** G is simple group of order 60. **(C)** $G'=\{e\}$ **(D)** $Z(G)=G$

Q.77. Inner automorphisms of $Q=\{\pm1,\pm i,\pm j,\pm k\}$ is _____.
(A) $\{e\}$ **(B)** $C_2\times C_2$ **(C)** Q **(D)** C_4

Q.78. Number of conjugacy classes of a cyclic group of order 6 is _____.
(A) 1 **(B)** 2 **(C)** 3 **(D)** 6

Q.79. Number of non-isomorphic abelian groups of order 12 is _____.
(A) 1 **(B)** 2 **(C)** 3 **(D)** 4

Q.80. Order of sylow-2 subgroup of Q_8 is _____.
(A) 1 **(B)** 2 **(C)** 4 **(D)** 8

Q.81. Which of the following is an ideal of R?
(A) Z **(B)** $\{0\}$ **(C)** C **(D)** Q

Q.82. Which of the following is not an integral domain?
(A) Z **(B)** Z_7 **(C)** Q **(D)** Set M_2 of 2×2 matrices with integer entries

Q.83. Which of the following is a field?
(A) $\{a+b\sqrt{2}:a,b\in Q\}$ **(B)** $Q\setminus\{0\}$ **(C)** Z **(D)** Z_6

Q.84. Which of the following is not a vector space?
(A) R(R) **(B)** R(Q) **(C)** R(C) **(D)** C(Q)

Q.85. Let $\varphi: Z\to Z_5$ be $\varphi(a)=a(\text{mod }5)$. Then $\text{Ker}(\varphi)=$ _____.
(A) $\{0\}$ **(B)** $\{0,\pm5,\pm10,\ldots\}$ **(C)** Z_5 **(D)** Z

Q.86. The number of proper ideals of Z_{17} is _____.
(A) 0 **(B)** 1 **(C)** 2 **(D)** 3

Q.87. Which of the following is a division Ring?
(A) $(Z,+,\cdot)$ **(B)** $(E,+,\cdot)$ **(C)** $(Q,+,\cdot)$ **(D)** $(Z_6,\oplus_6,\odot_6)$

Q.88. $\int_{-1}^{2}(x+|x|)\,dx=$
(A) 0 **(B)** 4 **(C)** 2 **(D)** 6

Q.89. $x=6$ in R^3 represents a
(A) point **(B)** Line **(C)** Plane **(D)** Space

Q.90. Kernel of $T:R^3\to R^3$, where $T(x,y,z)=(x,y,0)$, is
(A) Point **(B)** Line **(C)** Plane **(D)** Space

Q.91. Dimension of $\text{Hom}(R^3,R^4)=$
(A) 3 **(B)** 4 **(C)** 7 **(D)** 12

Q.92. Dimension of $\text{Hom}(M_{2,4},P_2(t))=$ _____.
(A) 4 **(B)** 8 **(C)** 16 **(D)** 24

Q.93. A dice is thrown. The probability that the dots on the top are prime numbers or odd numbers is _____.
(A) $\frac{1}{3}$ **(B)** $\frac{2}{3}$ **(C)** 1 **(D)** $\frac{5}{6}$

Q.94. A coin is tossed 4 times in succession. The probability that at-least one head occurs is _____.
(A) $\frac{1}{16}$ **(B)** $\frac{4}{16}$ **(C)** $\frac{12}{16}$ **(D)** $\frac{15}{16}$

Q.95. Number of necklaces made from 9 beads of different colors is _____.
(A) $\frac{8!}{2}$ **(B)** 8! **(C)** 7! **(D)** 9!

Q.96. Period of $3\cos\frac{x}{5}$ is _____.
(A) 2π **(B)** $\frac{2\pi}{5}$ **(C)** 6π **(D)** 10π

Q.97. Range of $\sec^{-1}x$ is _____.
(A) $[0,\pi]$ **(B)** $[0,\pi]\setminus\{\frac{\pi}{2}\}$ **(C)** $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$ **(D)** $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]\setminus\{0\}$

Q.98. Solution set of $\sin x\cos x=\frac{\sqrt{3}}{4}$ is _____.
(A) $\left\{\frac{\pi}{6}+n\pi\right\}\cup\left\{\frac{\pi}{3}+n\pi\right\}$ **(B)** $\left\{\frac{\pi}{3}+2n\pi\right\}\cup\left\{\frac{2\pi}{3}+2n\pi\right\}$ **(C)** $\left\{\frac{\pi}{6}+2n\pi\right\}\cup\left\{\frac{5\pi}{6}+2n\pi\right\}$ **(D)** $\left\{\frac{\pi}{12}+n\pi\right\}\cup\left\{\frac{5\pi}{12}+n\pi\right\}$

Q.99. Which of the following is tautology?
(A) $p\to\sim q$ **(B)** $(p\to q)\cap(p\ q)$ **(C)** $p\to q\leftrightarrow\sim q\to\sim p$ **(D)** $p\cap\sim p$

Q.100. $f(z)=\frac{1}{z}$ is not uniformly continuous in the region _____.
(A) $0\le|z|\le1$ **(B)** $0\le|z|<1$ **(C)** $0<|z|\le1$ **(D)** $0<|z|<1$